Chapter 1

> 说明：$\dot{x}$表示x对时间的一阶导数，$\ddot{x}$表示x对时间的二阶导数

1-1

解：

$$\begin{aligned} s &= x|_{t=4s} - x|_{t=0s} \\ &= 6\times 4 - 4^2 - 0 \\ &= 8m \end{aligned}$$

$$v = \frac{dx}{dt} = 6 - 2t(SI)$$

令$v=0$,有$t=3s.$

$$\begin{aligned} l &= |\Delta s_1| + |\Delta s_2| \\ &= 6\times 3 - 3^2 + 6\times 3 - 3^2 - 6\times 4 + 4^2 \\ &= 10m \end{aligned}$$

1-2

解：

$$\begin{cases} t = x/2 \\ y = 12 - 2t^2 \end{cases} (SI) \Rightarrow y = 12 - \frac{1}{2}x^2$$

$$\begin{cases} x = 2t \\ y = 12 - 2t \end{cases} (SI)$$

$\Rightarrow$

$$\begin{cases} \dot{x} = 2 \\ \dot{y} = -4t \end{cases} (SI)$$

$\Rightarrow$

$$\begin{cases} \ddot{x} = 0 \\ \ddot{y} = -4 \end{cases} (SI)$$

1-3

解:

$$s|_{t=4.5} = \int_0^{4.5} v(t)dt = 1 + 2 + 0.5 - 0.25 - 1 - 0.25 = 2m$$

1-4

解:

$$\begin{aligned} \frac{d^2x}{dt^2} &= 3 + 9x^2 \\ \Rightarrow v\frac{dv}{dx} &= 3 + 9x^2 \\ \Rightarrow vdv &= (3 + 9x^2)dx \\ \Rightarrow \int_0^t vdv &= \int_0^t (3 + 9x^2)dx \\ \Rightarrow \frac{1}{2}v^2 &= 3x + 3x^3 \\ \Rightarrow v &= \sqrt{6x + 6x^3} \end{aligned}$$

1-5

解:

- (1)

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{(9-16)-(4.5-2)}{1} = -9.5m/s$$

- (2)

$$v(t) = \frac{dx}{dt} = 4.5 - 6t^2$$

$$v(t=2) = 4.5 - 24 = -19.5m/s$$

- (3)
  当$t \in [1,2]$时，有$v < 0$，故

$$l = |\Delta x| = 9.5m$$

1-6

解:

$$\begin{aligned} \frac{dv}{dt} &= -kv^2 \\ \Rightarrow v\frac{dv}{dx} &= -kv^2 \\ \Rightarrow \frac{dv}{v} &= -kdx \\ \Rightarrow \ln v - \ln v_0 &= -kx \\ \Rightarrow v &= v_0 e^{-kx} \end{aligned}$$

1-7

解:

- (1)

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = -a\omega \sin \omega t\boldsymbol{i} + b\omega \cos \omega t\boldsymbol{j}$$

- (2)

$$\begin{cases} x = a\cos \omega t \\ y = b\sin \omega t \end{cases}$$

  为椭圆的参数方程。
- (3)

$$\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = -a\omega^2 \cos \omega t\boldsymbol{i} - b\omega^2 \sin \omega t\boldsymbol{j}$$

  有$\boldsymbol{a} = -\omega^2\boldsymbol{r}$，即$\boldsymbol{a}$与$\boldsymbol{r}$方向相反。由(2)有$\boldsymbol{r}$背离中心，故$\boldsymbol{a}$指向中心。

1-8

解:

$$\begin{aligned}\frac{x}{H} &= \frac{x-s}{h}\\ \Rightarrow hx &= Hx - Hs\\ \Rightarrow (H-h)x &= Hs\\ \Rightarrow x &= \frac{Hs}{H-h}\\ \Rightarrow v_{head} &= \frac{Hv_0}{H-h}\end{aligned}$$

1-9

解:
有物体加速度为$\boldsymbol{a} = -g\boldsymbol{j}$。
分解到切向与法向，有:

$$\begin{cases} a_{\parallel} & = -g\sin\theta \\ a_{\perp} & = -g\cos\theta \end{cases}$$

1-10

解:

质点速度为:

$$v = \frac{ds}{dt} = b - ct$$

质点加速度为:

$$\begin{cases} a_t = |\dot{v}| = c \\ a_n = v^2/R = (b-ct)^2/R \end{cases}$$

令$a_t = a_n$，有:

$$t = b/c \pm \sqrt{R/c}$$

1-11

解:

- (1)
  切向速度v:

$$v=\frac{ds}{dt}=v_0-bt$$

  切向加速度a_t与法向加速度a_n:

$$\begin{cases}a_t=\dot{v}=-b\\ a_n=v^2/R=(v_0-bt)^2/R\end{cases}$$

- (2)
  加速度大小a:

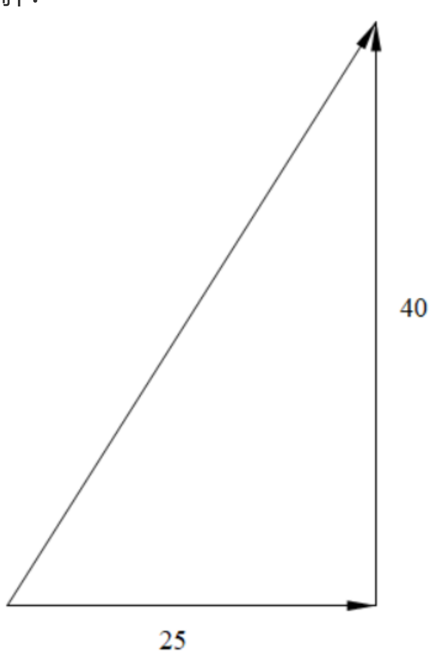
$$\begin{aligned}a&=\sqrt{a_t^2+a_n^2}\\&=\sqrt{b^2+a_n^2}\end{aligned}$$

  令$a=b$，有:

$$a_n=0\Rightarrow v_0=bt\Rightarrow t=\frac{v_0}{b}$$

1-12

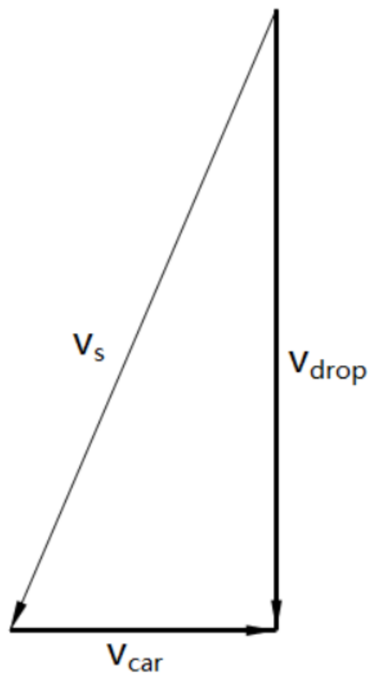
解:



$\boldsymbol{v}=25km\cdot h^{-1}\boldsymbol{i}+40km\cdot h^{-1}\boldsymbol{j}$

1-13

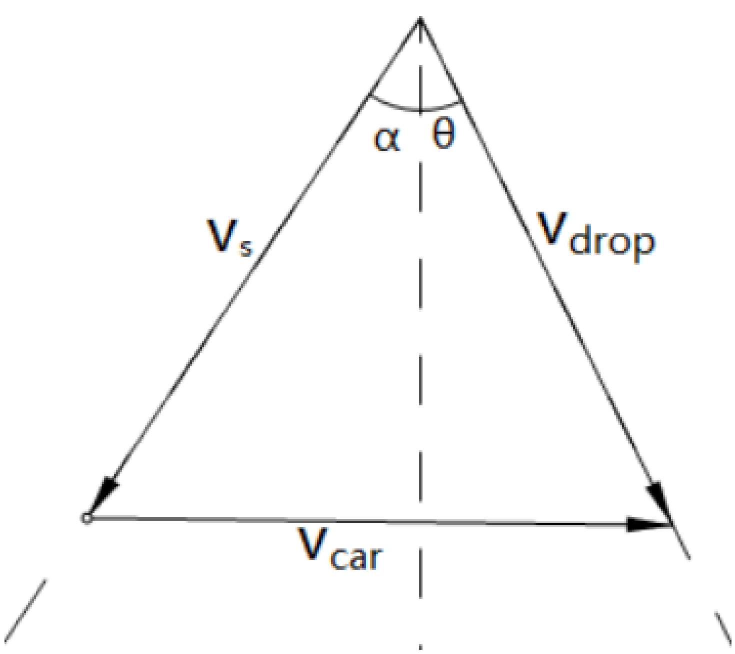
解:



$v_s = \sqrt{v_{car}^2 + v_{drop}^2} = 4\sqrt{58}$，方向向下偏北$\arctan\frac{3}{7}$

1-14

解:



$\tan\alpha = \frac{l}{h}, v_1 = v_2\sin\theta + v_2\cos\theta\tan\alpha$

$\Rightarrow v_1 = v_2(\sin\theta + \frac{l}{h}\cos\theta)$

1-15

解:
有河流流速为:

$$v_{water} = \begin{cases} \frac{2v_0}{L}y, & y \in [0, \frac{L}{2}] \\ \frac{2v_0}{L}(l-y), & y \in [\frac{L}{2}, L] \end{cases}$$

1. 向河中心行驶:
   有$y = ut, v_x = \frac{2v_0}{L}y$。
   可得:

$$\begin{aligned} v_x &= \frac{2v_0 u}{L}t \\ \Rightarrow x &= \frac{2v_0 u}{L}\int_0^t tdt \\ \Rightarrow x &= \frac{v_0 u}{L}t^2 \end{aligned}$$

   末状态有$t_1 = \frac{L}{4u}, x_1 = \frac{v_0 L}{16u}$,轨迹方程为 $x = \frac{v_0}{uL}y^2$
2. 回程:
   有$\frac{L}{4} - y = \frac{u}{2}(t - t_1), v_x = \frac{2v_0}{L}y$。
   可得:

$$\begin{aligned} v_x &= \frac{3v_0}{4} - \frac{v_0 u}{L}t \\ \Rightarrow x - x_1 &= [\frac{3v_0}{4}t - \frac{v_0 u}{2L}t^2]_{t_1}^{t} \\ \Rightarrow x &= \frac{3v_0}{4}t - \frac{v_0 u}{2L}t^2 - \frac{3v_0 L}{32u} \end{aligned}$$

   末状态有$t_2 = \frac{3L}{4u}, x_2 = \frac{3v_0 L}{16u}$,
   轨迹方程为 $x = -\frac{2v_0}{uL}y^2 - \frac{v_0}{u}y - \frac{3v_0 L}{8u} + \frac{9}{16}v_0 L$。

综上:
轨迹方程:

$$x = \begin{cases} x = \frac{v_0}{uL}y^2, & x \in [0, \frac{v_0 L}{16u}] \\ x = -\frac{2v_0}{uL}y^2 - \frac{v_0}{u}y - \frac{3v_0 L}{8u} + \frac{9}{16}v_0 L, & x \in [\frac{v_0 L}{16u}, \frac{3v_0 L}{16u}] \end{cases}$$

返回本岸时离出发点的距离为$\frac{3v_0L}{16u}$。